Exercise 12

Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$$g(t) = \frac{t^2 + 5t}{2t + 1}, \quad a = 2$$

Solution

By definition, a function is continuous at a number a if

$$\lim_{t \to a} g(t) = g(a).$$

Evaluate the function at $t = 2$.

$$f(2) = \frac{(2)^2 + 5(2)}{2(2) + 1} = \frac{4 + 10}{4 + 1} = \frac{14}{5}$$

Calculate the limit as t approaches 2 using the limit laws.

$$\begin{aligned}
\lim_{t \to 2} g(t) &= \lim_{t \to 2} \frac{t^2 + 5t}{2t + 1} \\
&= \frac{\lim_{t \to 2}(t^2 + 5t)}{\lim_{t \to 2}(2t + 1)} \\
&= \frac{\lim_{t \to 2} t^2 + \lim_{t \to 2} 5t}{\lim_{t \to 2} 2t + \lim_{t \to 2} 1} \\
&= \frac{\left(\lim_{t \to 2} t\right)\left(\lim_{t \to 2} t\right) + 5\left(\lim_{t \to 2} t\right)}{2\left(\lim_{t \to 2} t\right) + 1} \\
&= \frac{(2)(2) + 5(2)}{2(2) + 1} \\
&= \frac{14}{5}
\end{aligned}$$

The condition in the definition is satisfied, so $g(t) = \frac{t^2+5t}{2t+1}$ is a continuous function at $a = 2$.